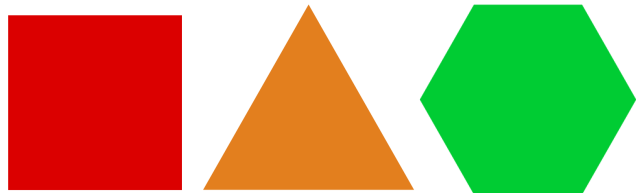# Tilings for kindergarten and primary level

Participants:
>Age: age 4 and up
>No previous mathematical knowledge is required.

Preparations:
>Print the **tiling sheets** on thick colored paper or thin cardboard and cut out the tiles. Depending on the age of the participants, you might want to start with the simpler shapes (square or rectangle). A cutting knife could help to make precise and quick cuts - please make sure to assist younger participants. You can also print on different colors to create unique combinations.

Activity 0:
>The teacher distributes the tiles (section 1 of the tiling sheet) among the participants and asks them to place the tiles of a given shape next to each other without leaving any space between them and without the tiles overlapping (in mathematical language, we call this a "tiling of the plane"). The tiling sheet provides some solutions.

Activity 1:
>Can you guess the conditions on the angles that make it possible or impossible? For instance, there is exactly one shape for which the tiling is impossible. Can you guess which one?

Activity 2:
>Look at the tilings of the plane you have produced and convince yourself that you could continue it infinitely in any direction. Notice that the tiling creates a pattern that repeats itself in two directions. In any of these directions, there is a distance such that if you move the tiling by this distance in the given direction, then you can superimpose it on the original tiling (of course, if we imagine the tiling to be infinite).

Activity 3:

Flip some of the shapes on their back side and try to superimpose them with the unflipped shapes. We say that the flipped shape is the *mirror image* of the original shape. For which shapes is the mirror image not superimposable? For these shapes, is it possible to tile the plane using copies of both the shape and its mirror image?

Activity 4:

This new shape called the *hat,* is an *Einstein tile* (find it in the tiling sheet). Verify that it cannot be superimposed on its mirror image. The hobbyist David Smith discovered in November 2022 that it is possible to tile the plane with copies of both this shape and its mirror image.

The Einstein tile   Its mirror image
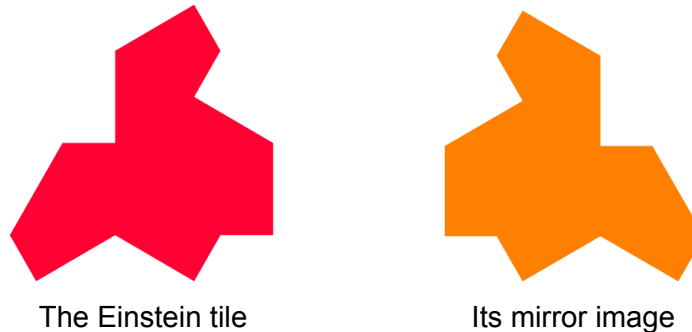
A second Einstein tile called the *vampire* was announced in May 2023. This vampire can tile the entire plane without using mirror images. It was created by David Smith, Joseph Samuel Myers, Craig S. Kaplan and Chaim Goodman-Strauss.

Solution: Examine the hat tiling and notice that we have both hat tiles and their mirror images: choose one reference tile and examine which other tiles can be superimposed on it and which ones cannot. Note also that the pattern is not repeated in any direction.

With the vampire tile, two types of patterns can be produced: patterns that repeat in two directions if we allow both the tile and its mirror image, and patterns that do not if we forbid mirror images. The tiling sheet shows a pattern that does not repeat.

Create and Share!
You can make tilings with other shapes or with multiple types of shapes. Share the tessellations you have created using the hashtags **#idm314paper** and **#idm314**.

Reference
This reference contains the Einstein tiles.