Group screening

Participants:

Ages 12 and up, divided into groups of 4-5 people.

Overview:

Controlling a pandemic requires a lot of tests. Testing is expensive and requires a lot of reactants, while most of the tests are negative. Can we do better? For instance, suppose that we mix samples from 12 individuals and test the mix. If that single test on the mixed sample is negative, we can conclude that the 12 individuals are negative. If the test is positive, an additional round of testing is necessary. We could test each of the 12 individuals or divide the group into four subgroups of 3 people and test a mix of samples of each subgroup. In the second case, a third round of testing is necessary. This simple idea is the basis for the “group screening” method (also called “pooled testing”), which dates back to World War II where it was used to test US army drafts for syphilis. The activities below explore the method.

Activity 1

- Start by describing the principle.
- Suppose we need to test 100 individuals, numbered 1 to 100. Pick two numbers at random, representing the two infected individuals. Each team chooses one member to be the referee. Secretly tell the referees who the two infected are. The other students of the team have to find the infected through successive rounds of testing.
- The team proposes how to group and test the individuals. The referee tells which groups got a positive test. The process is repeated through several rounds until the two infected are found. Each team counts the total number of tests it needed.
- Share the strategies used by the different teams. Were some strategies better than others?
- Nominate a different referee and repeat the game with only one infected individual among 100.
- Nominate a different referee and repeat the game with three infected individuals among 100.
- Share the strategies used by the different teams. Were some strategies better than others?

Activity 2

If each round of tests takes some time, it's essential to minimize the number of rounds.

- Suppose we need to test 100 individuals, numbered 1 to 100. Pick two numbers at random, representing two infected individuals. Each team chooses one member to be the referee. Secretly tell the referees who the two infected are. The other students of the team have to find the infected through, at most, two rounds of testing.
- The team proposes how to group and test the individuals, and the referee tells which groups got a positive test. The team only has two rounds of testing to find the two infected this time. Each team counts the total number of tests it needed.
- Share the strategies used by the different teams. Were some strategies better than others?

Activity 3

Sometimes it's possible to use only one round of tests.

- Suppose we need to test 16 individuals. Can any team propose a strategy to test them and find up to two infected individuals in a single round?
- Here is a strategy: We'll call the 16 people {A, B, C, …, P} and do 12 tests. Each line in the following table represents a single test. The Xs indicate the individuals whose samples are mixed for each test. For instance, for test 1, we mix samples from A, B, C, and D. Note that each person's sample is used in three different tests.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
Test 1	X	X	X	X												
Test 2					X	X	X	X								
Test 3									X	X	X	X				
Test 4													X	X	X	X
Test 5	X				X				X				X			
Test 6		X				X				X				X		
Test 7			X				X				X				X	
Test 8				X				X				X				X
Test 9	X					X					X					X
Test 10		X					X					X	X			
Test 11			X					X	X					X		
Test 12				X	X					X					X	

- Show that this method allows finding all infected if at most two persons of the group are infected.
- Show that if three persons or more are infected, this single-around algorithm can't identify them.

Activity 4

Here's another algorithm using a single round of tests.

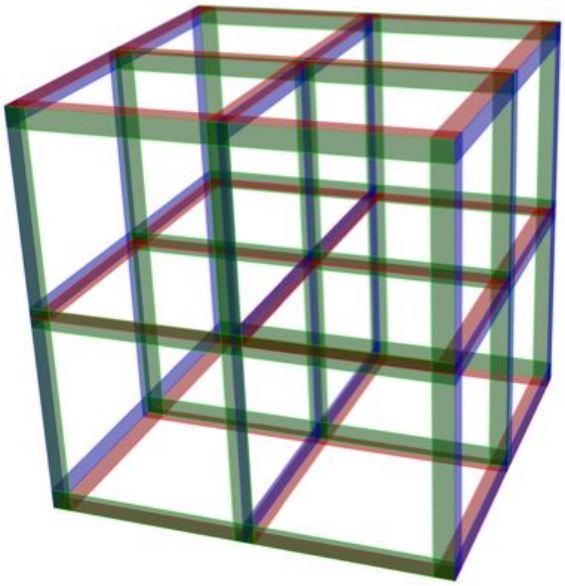
- We place n^2 people in the squares of an n×n grid. Group screening goes as follows: We mix samples from each horizontal row and each vertical column (hence, we perform 2n tests).
- Show that if only one person is infected, we can identify it with a single round of tests.
- Show that if there's more than one infected person, but they are all in the same horizontal row or vertical column, we can identify them with a single round of tests.
- Suppose that two lines and two columns have a positive test. What are the minimum and maximum possible number of infected people? How many additional tests do you need to perform in a second round of tests to detect all infected people?
- Suppose that there are three infected individuals. Describe all possible couples (r,s) where r is the number of rows testing positive and s is the number of columns testing positive.

Here's a visual explanation of this method, with two infected among 36 people.

Activity 5

The following algorithm is more advanced. It uses the same idea as Activity 4 but in a higher number of dimensions. Hence, you can use it with advanced students. It was proposed by a Rwandan team of researchers and used for COVID-19 testing.

- The algorithm works for groups of 3^m individuals. It consists of performing 3m tests simultaneously on mixtures of samples, each containing 3^{m-1} individuals. The 3^m individuals are identified with the points $\{0, 1, 2\}^m$ of an m-dimensional cube (see the figure below for m = 3). The pools are slices of the hypercube. Indeed, if $x_1, \ldots, x_m$ denote the coordinate axes of the hypercube, each of the mixtures corresponds to the individuals located in the hyperplane $x_i = t$, where $i \in \{1, \ldots, m\}$ and $t \in \{0, 1, 2\}$ is a slice of 3^{m-1} individuals (the red, green, and blue slices in the figure).



- The Rwandan team of researchers proposes to use the algorithm for m = 4, i.e., 12 tests per group of 81 individuals.
- Show that this round of tests is sufficient to identify exactly one infected individual.
- Determine the number of additional tests needed in a second round of testing if two individuals are infected. You will need to consider three cases: i) the two infected individuals lie on a line parallel to the coordinate axes; ii) the two infected individuals lie inside a test slice but not on a line; iii) the general case.

Further resources

- An article about group screening, by Christian Genest and Christiane Rousseau.

Create and Share!
Share pictures and videos of the activity or strategies proposed by the group, using the hashtag **#idm314**.