Artistic Tilings

This activity offers step-by-step guides for making artistic tile patterns. After understanding how these designs work, you'll be encouraged to create unique patterns.

Participants:

Ages 8 and up, depending on the activity.
No prior math knowledge is needed, but you may come across some math concepts as you progress through the activities. These could include shapes like polygons, triangles, quadrilaterals, and hexagons, as well as ideas like parallel sides, translation, rotation, and line symmetry.

Preparations:

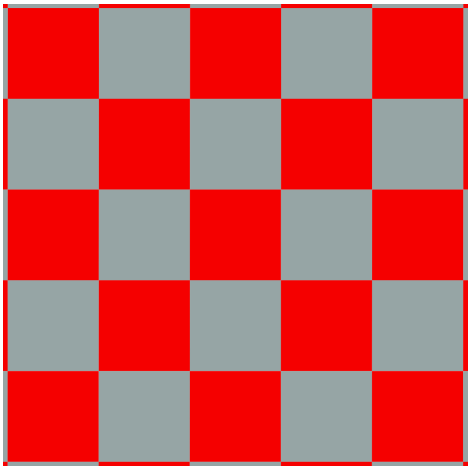
Paper, pencil, eraser, color pencils or scissors, and color paper.
If the children have access to tablets or computers and the Internet, they can create patterns on some software (for instance, https://tiled.art, which is free).

Activity 1: Tilings using only translations.[1]

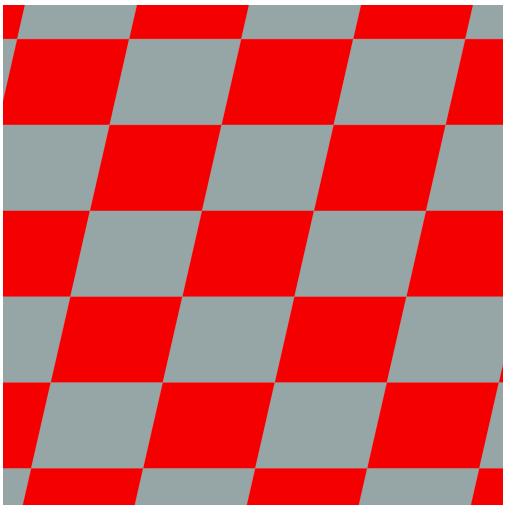
a. Quadrilateral tiling. **(Ages 8 and up)**

The first step is to recognize that a square (4 sides) can cover the entire plane in a repeating pattern. This means we can arrange square tiles to completely fill the plane without any gaps or overlapping tiles.

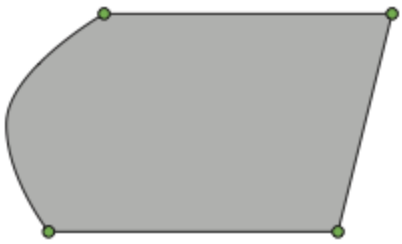
[1] The tilings of this activity have been realized with the tool https://tiled.art



Note that the tiles have pairs of parallel and equal sides. In these tilings, the corners of the tiles are aligned and connected to each other. In all the activities below, we will focus on tilings where the corners of the tiles are always glued together.
The squares can be transformed into irregular shapes with the same number of sides and still cover the plane in a repeating pattern, as long as the parallel sides stay parallel and equal in length. The resulting shape is called a *parallelogram*.

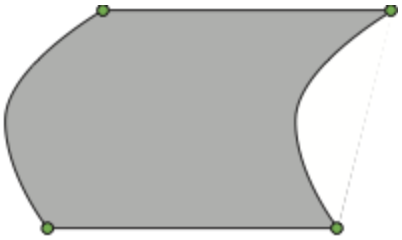


Now, begin by creating such a tiling and select a gray tile. Take one side of the tile, such as the left side, and curve it, ensuring the endpoints of the curve remain the same as the original straight side.

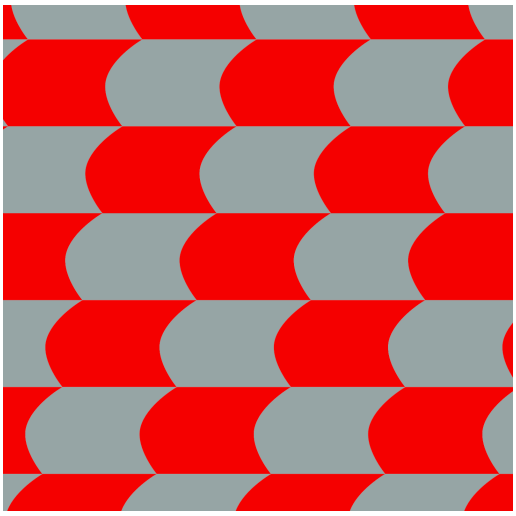


It is no longer possible to tile the plane with these tiles because the left side of one tile no longer matches the right side of another tile. To make it possible to tile the plane again, the same

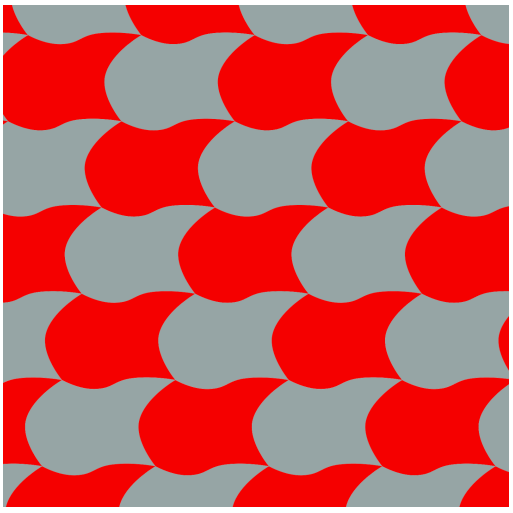
deformation must be applied to the right side, while ensuring the endpoints stay the same as well.



Now this gives a new tiling:

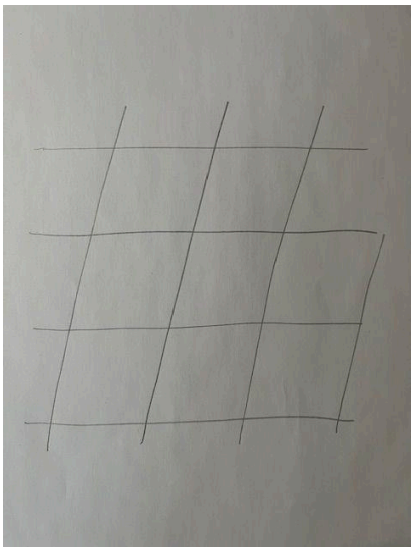


You can do the same with the sides in the other direction, meaning you replace them with curves that have the same endpoints.

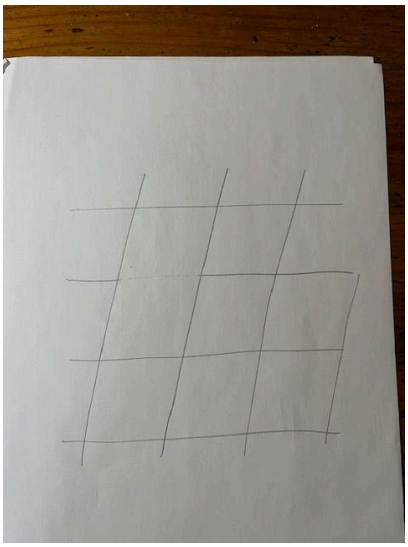


- Explain why this recipe works.
- Create your own patterns. Start by drawing the initial tiling with a pencil, and use an eraser to adjust the straight lines into curves.

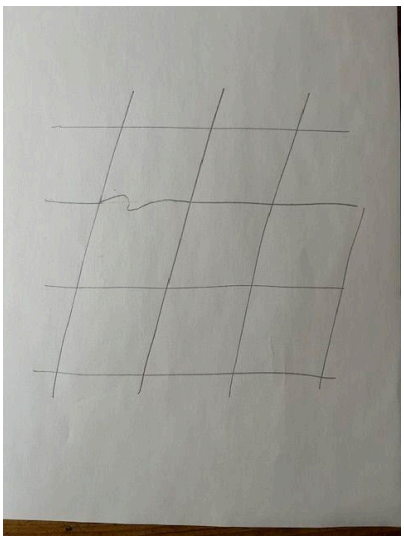
Start with a grid,



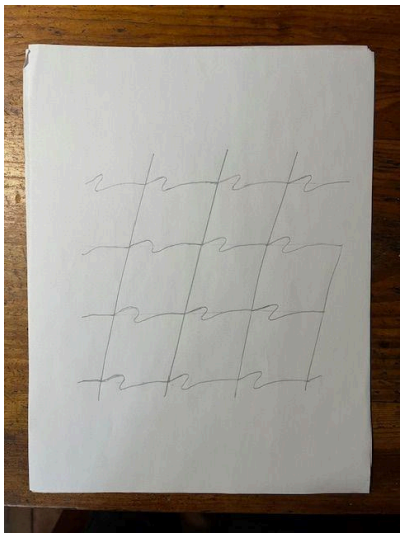
Then erase one side.



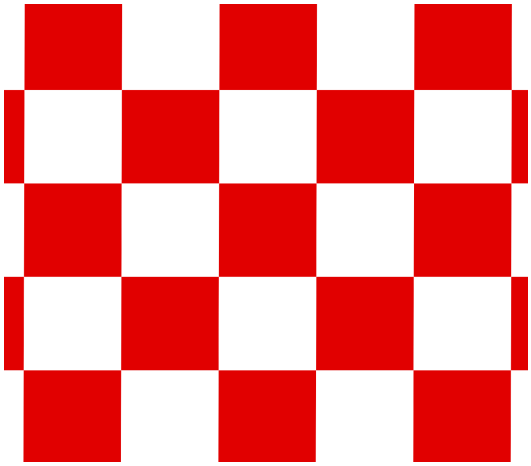
Replace it by any curve with the same endpoints.

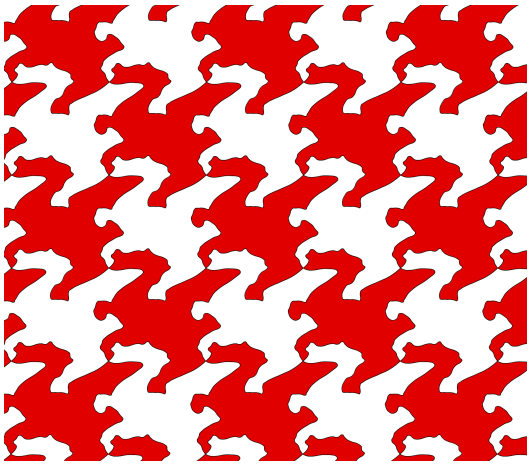


Then replace all parallel segments with the same curve.



Repeat with the other direction.
Afterwards, you can color the tiles. Alternatively, you can cut out tiles from sheets of colored paper using scissors and assemble them. Here is an example of a tiling featuring flying horses, inspired by the artist M.C. Escher's print, where the original tiles are squares.

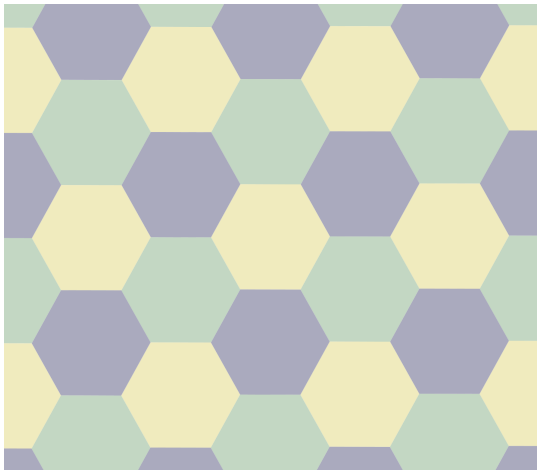




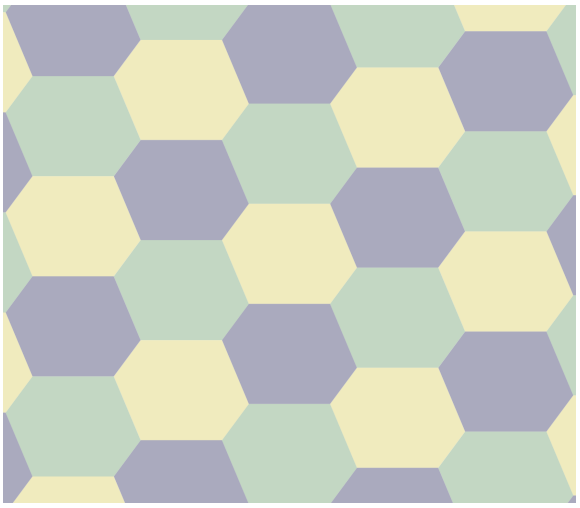
- Can you create a pattern where the tiles are shaped like something from nature (such as a plant, animal, or part of a plant) or a recognizable object?

b. Hexagonal tiling. **(Age 12 and up)**

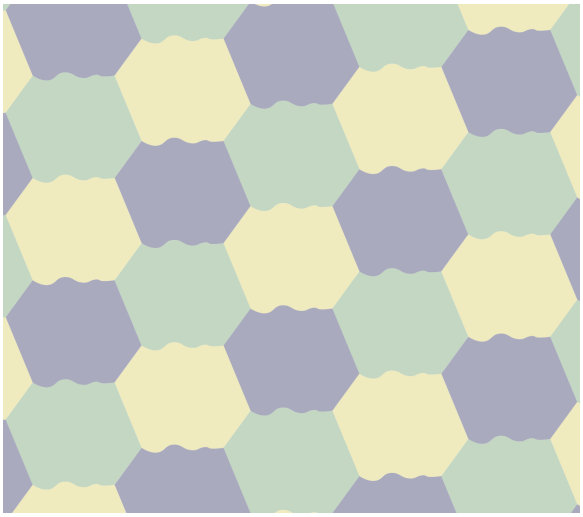
Here, the regular *hexagon* (6 sides) can cover the entire plane in a repeating pattern.



Note that each tile has three pairs of parallel and equal sides, and once again, the corners of the tiles are aligned and connected. These regular hexagons can be transformed into irregular hexagons (polygons with 6 sides) and still tile the plane in a repeating pattern, as long as the parallel sides stay parallel and of equal length.



Now, take any side of a tile and curve it, keeping the endpoints the same as the original straight side. If you want to tile the plane with this new tile, you must follow the same rule as before: apply the same deformation to the side parallel to the one you just curved. This will result in a new tiling pattern.



Now that there are three pairs of parallel sides, you can apply the same deformation to the other two sets of parallel sides, curving them in the same way while keeping the endpoints fixed. This will create a new, deformed tiling pattern

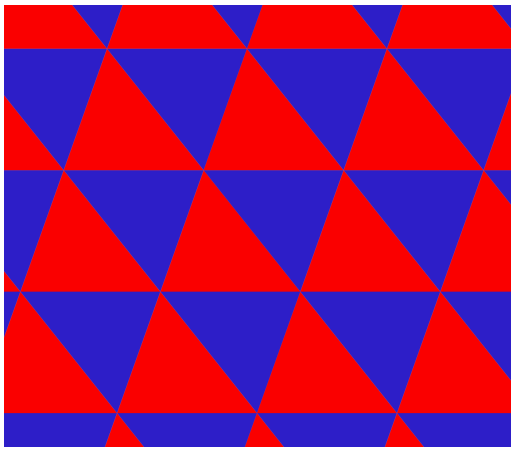
- Explain why this recipe works.
- Create your own patterns. Start by drawing the initial tiling with a pencil, and use an eraser to adjust the straight lines into curves. Afterward, you can color the tiles. Alternatively, you can cut out tiles from sheets of colored paper using scissors and assemble them.
- Can you create a pattern where the tiles are shaped like something from nature (such as a plant, animal, or part of a plant) or a recognizable object?

Activity 2: Tilings with 180-Degree Rotations.[2]

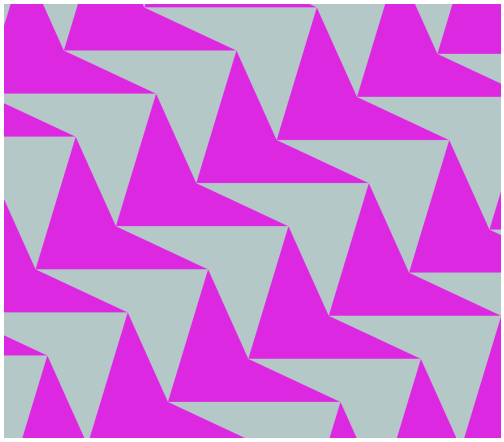
c. *Triangular or quadrilateral tiling.* (Ages 12 and up)

The first step is to realize that any triangle or quadrilateral (a polygon with four sides) can tile the plane if 180-degree rotations of the tiles are allowed, in addition to translations.

[2] The tilings of this activity have been realized with the tool https://tiled.art.



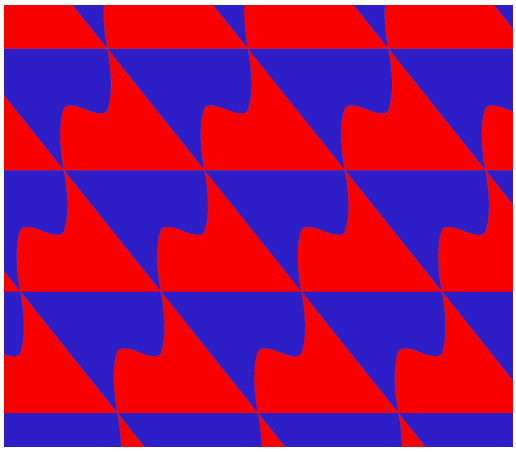
Tiling with triangles

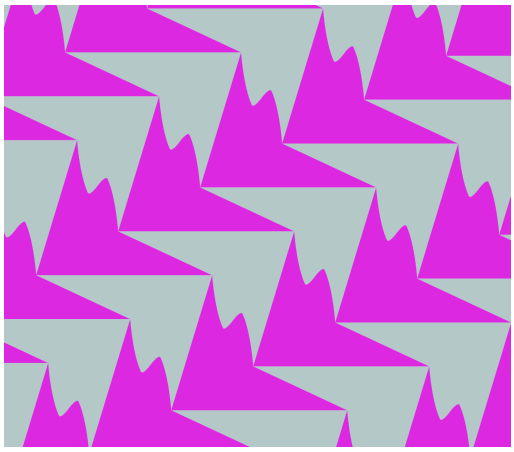


Tiling with quadrilaterals

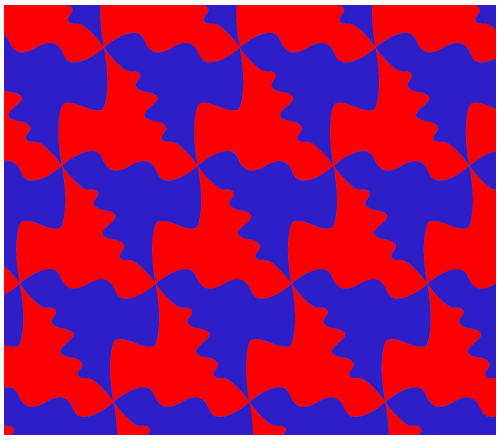
Now, start with any of these tilings, pick a tile, and choose one of its sides. Deform this straight side into a curve, making sure the endpoints stay the same.

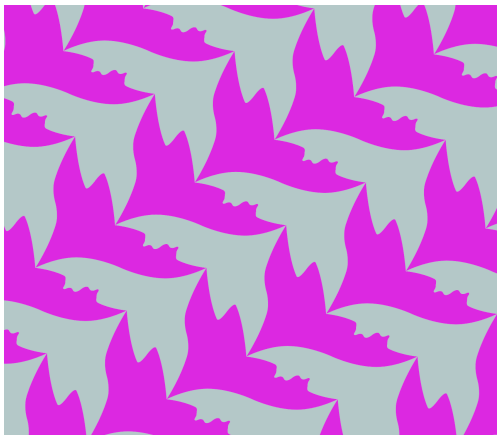
- Be careful: you can't change the side randomly if you want the tiles to still fit together to cover the plane. The curved side needs to match the rotated curved side of the tile next to it. This means the side must look the same if you rotate it 180 degrees around the middle point of the side. In other words, you can't move the middle point of the side. Following this rule will give you a new tiling pattern.





You can do the same with the other sides in different directions. Just apply the same rule: curve the sides while making sure they match the rotated sides of the adjacent tiles.

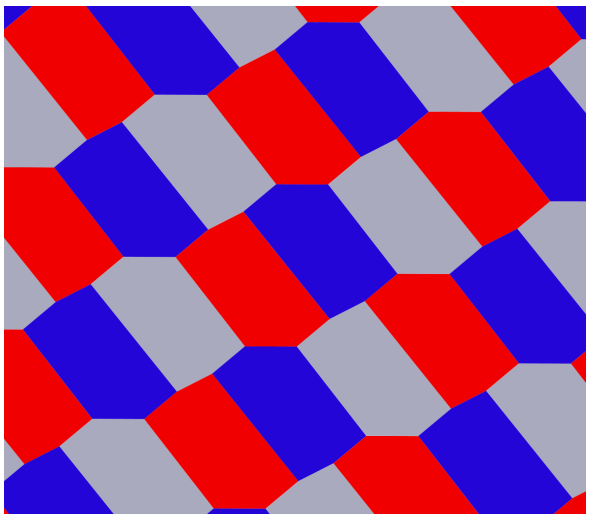




- Can you explain why this recipe works and why the curves replacing the sides need to be symmetric around their center?
- Create your own patterns.
- Can you create a pattern where the tiles are shaped like something from nature (such as a plant, animal, or part of a plant) or a recognizable object?

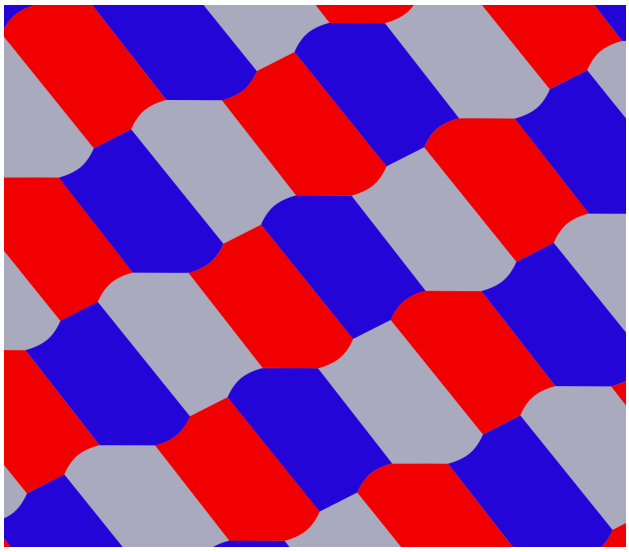
d. Hexagonal tiling. **(Age 14 and up)**

One way to create such tilings starts with hexagons (shapes with six sides) that have one pair of parallel opposite sides with the same length.[3]

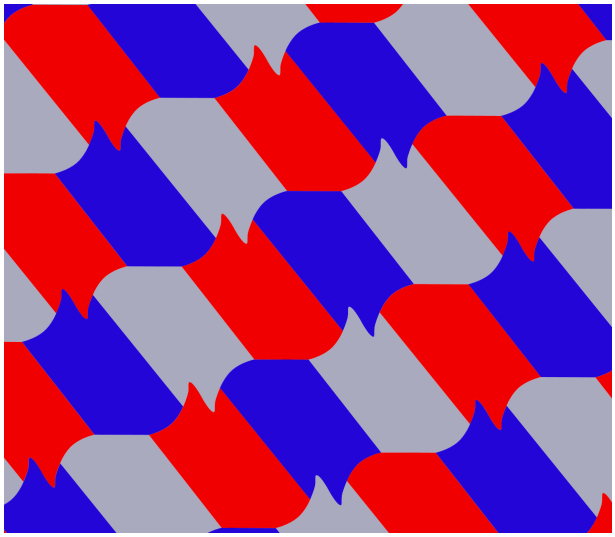


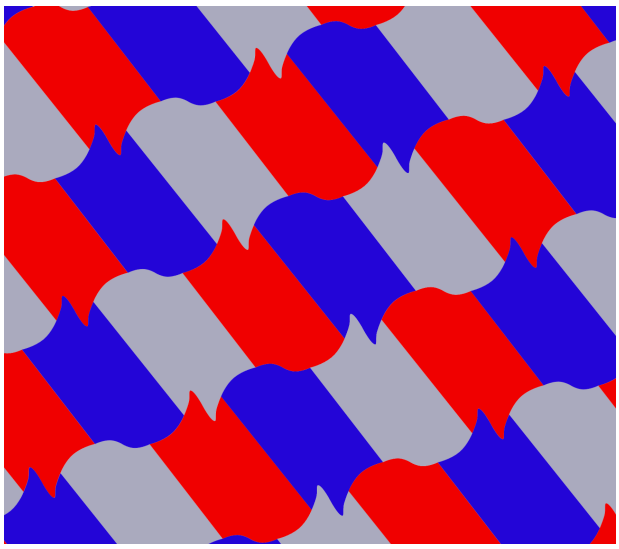
The parallel and equal sides can be replaced with any two parallel and equal curves, as long as the endpoints remain the same.

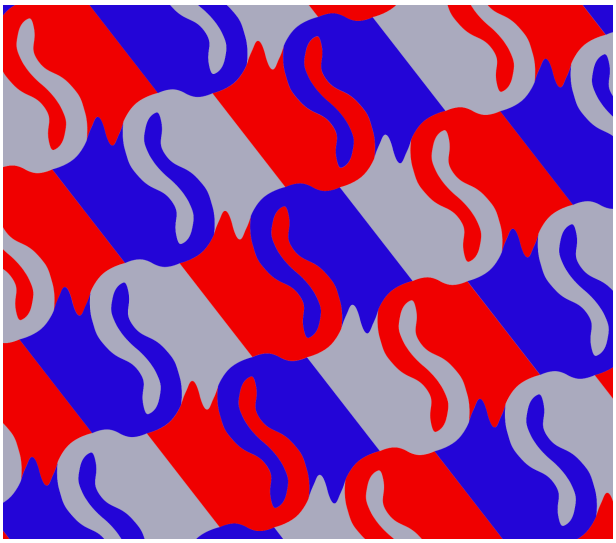
[3] This method was provided by the mathematician John Conway.

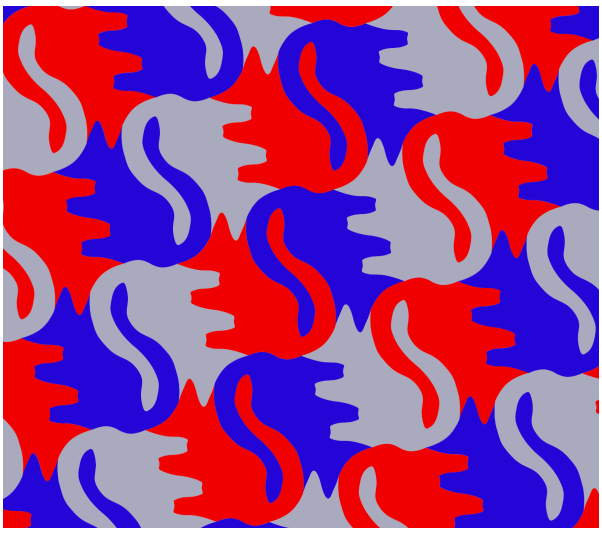


Each of the other four sides can be replaced with any curve that is symmetric around the center of the side while keeping the endpoints the same. Here are the steps to follow:









- Explain the conditions of this method.
- In the original hexagonal tiling, three hexagons meet at each corner. This is why the tiling can be colored using three different colors.
- Create your own tilings.

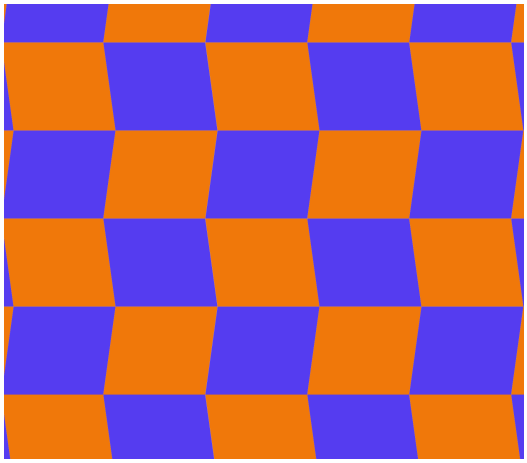
Activity 3: Tilings with flips.[4] (Ages 14 and up)

We will use tiles that can cover the whole plane, with half of them flipped (like making a mirror image along a line) along with translations. There are two types of four-sided shapes (quadrilaterals) where this works: *parallelograms*, where opposite sides are parallel, and *kites*, which have two pairs of equal adjacent sides.

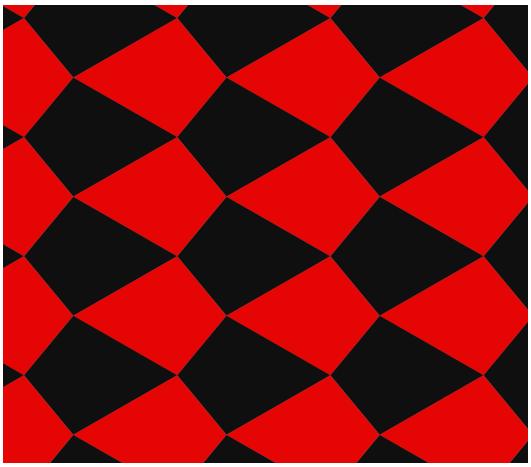
For parallelograms, each pair of parallel sides is called a "twin pair." For kites, each pair of equal sides is also called a "twin pair."

Parallelograms and kites make tilings as in the following images:

[4] The tilings of this activity have been realized with the tool https://tiled.art.



Tiling with parallelograms



Tiling with kites

Note that in both cases, there are parallel symmetry lines (horizontal in the pictures), we will call their direction the *flip direction*. The perpendicular direction (vertical in the pictures) will be called the *glide direction*.

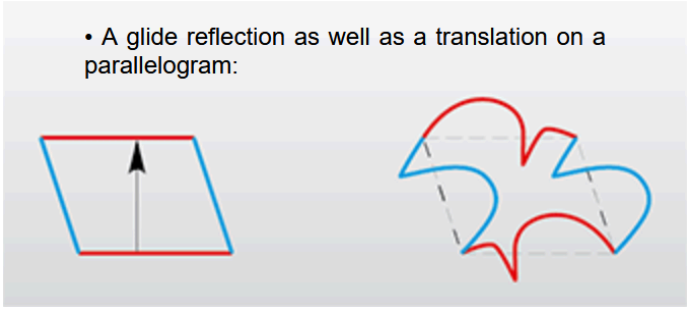
In any case, when you deform one side into a curve, you must also deform its twin side correspondingly. To do so there are two possible transformations:

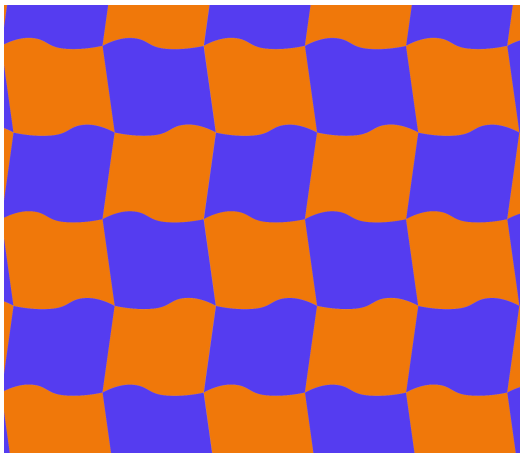
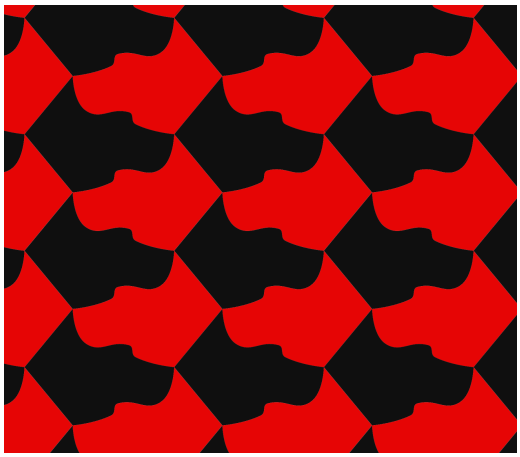
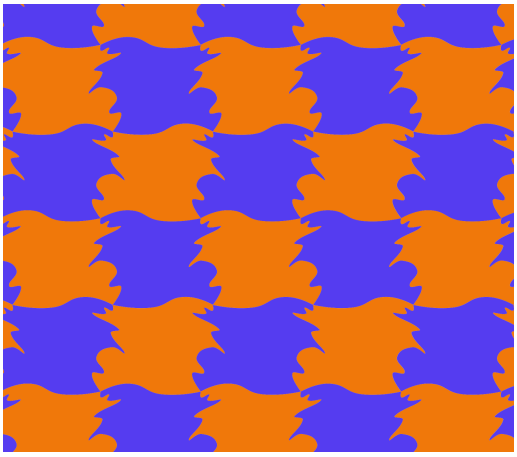
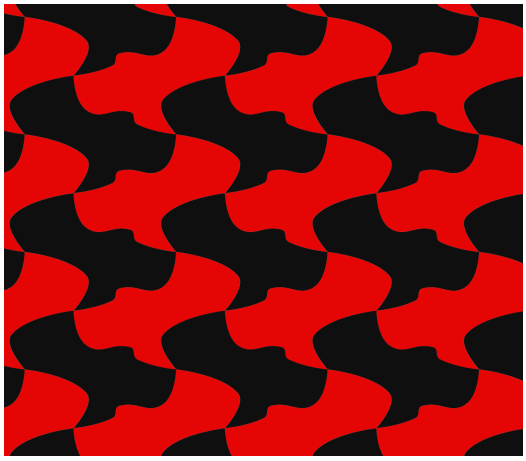
- Translation along a direction
- Glide reflection along a direction. A glide reflection is a reflection followed by a translation (both in the same direction)

If you pick a parallelogram and you deform a side parallel to the flip direction, you modify its twin by a glide reflection on the glide direction.

If you pick a parallelogram and you deform a side that is not parallel to the flip direction, you modify its twin by a translation on the symmetry direction.

If you pick a kite, any side that you deform, you must modify the corresponding twin side by a glide reflection on the glide direction.



- Discuss why these rules work.
- Create your own patterns.

Activity 4: Tilings with other types of symmetries. (Ages 14 and up)

There are many different ways to create tilings using shapes that repeat in a pattern, with different types of symmetries. You can try making your own by experimenting with shapes and symmetries, or you can check out some examples on this website: https://tiled.art**.**

Mathematical background and resources:
https://tiled.art
https://en.tessellations-nicolas.com/method.php

Create and Share!
Share the participants' findings using the hashtags **#idm314tilings** and **#idm314**.