The Art Gallery (primary level)

Participants:

Ages 8 and higher.
No previous mathematical knowledge is required.

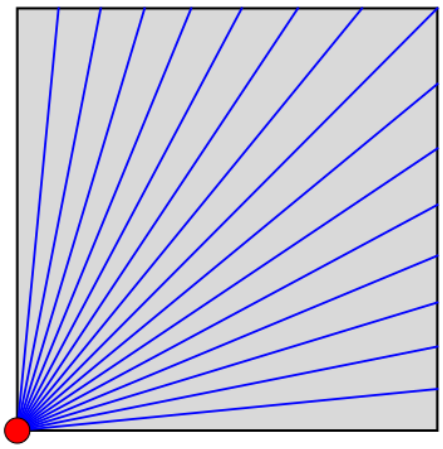
Preparations:

Printed templates, and pencils of three different colors.
Blank paper sheets to create your own galleries.
Another possibility is to play the activity on the street with colored chalk.

Activity: Protecting the gallery with cameras

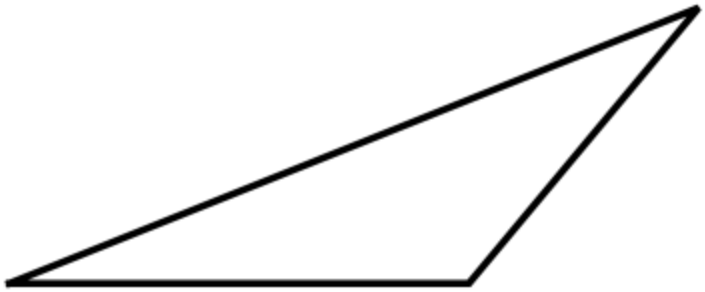
Imagine you are in an art gallery with amazing artwork. The floor plan of the gallery is shaped as a rectangle or can have a very fancy shape with lots of corners and twists. Your mission is to strategically place cameras at specific spots in the gallery to make sure that every spot in the gallery can be seen and is under surveillance. But here's the catch: you have to use as few cameras as possible. And these cameras can only be placed at the corners of the gallery.

In the picture below, we have a gallery in the form of a square, the red dot marks a camera. Using a pencil, you can draw straight lines starting from the camera to trace the area the camera can see.

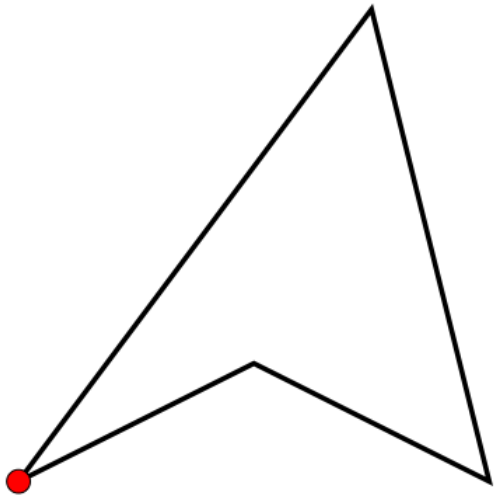


Remember that it cannot see through walls. You can also trace the camera's line of sight with a ruler to see what the camera can cover. You can see that the whole gallery is under surveillance with only one camera.

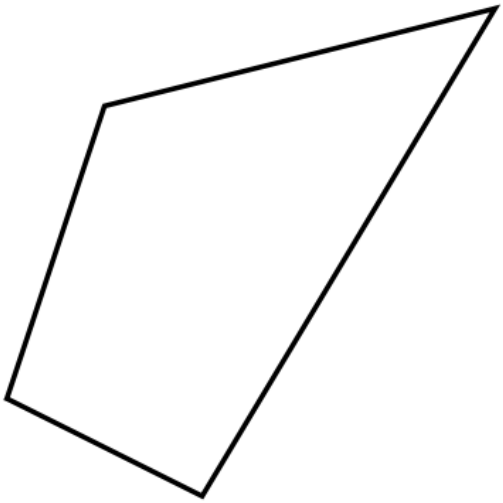
- What kind of fancy shapes for a gallery can you think of? The straight lines, which are the *sides* of the shape and also the walls in our gallery, are not allowed to cross and can only meet in the corners. Also, they connect to form a complete shape with no gaps. Draw shapes with 3 sides, 4 sides, 5 sides, 6 sides.
- Explore that all of your shapes have an inside part and an outside part. Color the insides of your shapes.
- Start discussing what it means that a certain point – for example, a very expensive painting – is watched by a camera: it means that there are no obstacles between the painting and the camera. If a person standing in front of that painting is watched by the camera, then that person can see the camera. In the art gallery, a painting is watched by a camera if you can draw a straight line segment between the painting and the camera, and the whole line segment is located inside the gallery.
- Now, your art gallery has the shape of a triangle. Experiment that one camera is sufficient to watch the whole gallery and that it can be placed at any corner of the triangle.



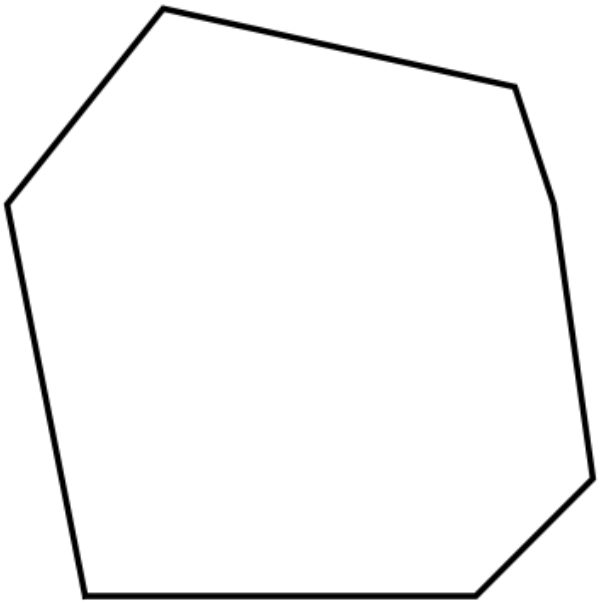
- The following gallery has four sides. Draw the region watched by the camera (red dot) in this gallery. Can it watch the whole gallery? Can you move the camera to another corner so that it watches the whole gallery?



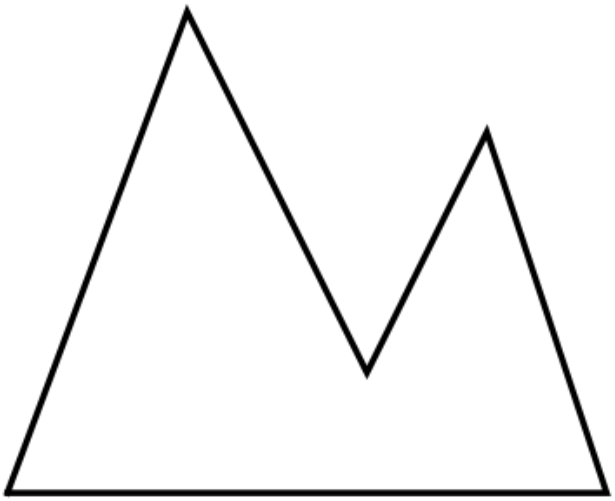
- The next gallery is also one with four sides. At which corners can you place one camera so that it watches this whole gallery?

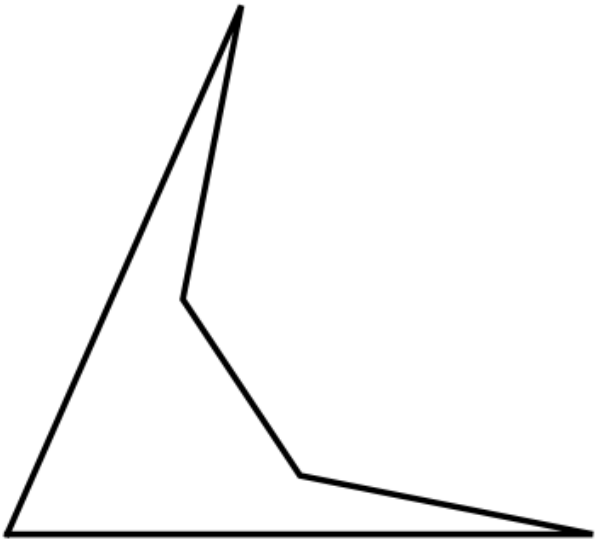


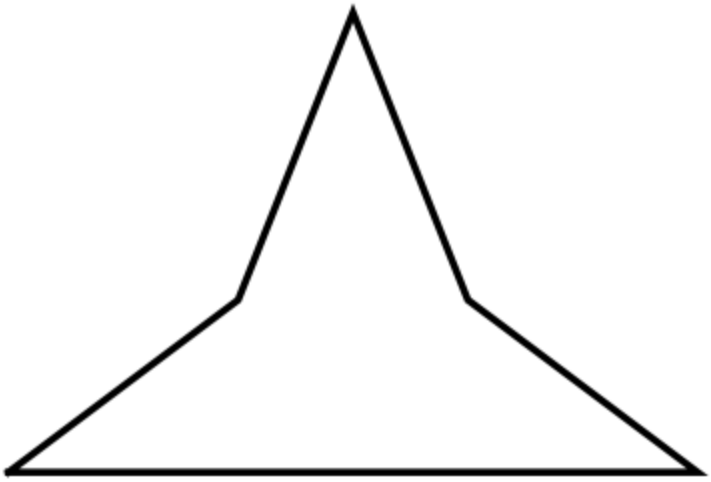
- Can you explain the difference between the two galleries with four sides? In the first case, there are points in the gallery that you can not connect with a straight line segment without crossing the sides/walls of the gallery: we say that the shape is *non-convex*. In the second case, any two points in the gallery can be connected by a straight line inside the gallery: we say that the shape is *convex*.
- For any convex gallery, **only one camera suffices** and it can be placed at any corner. Is this the case for the following gallery?



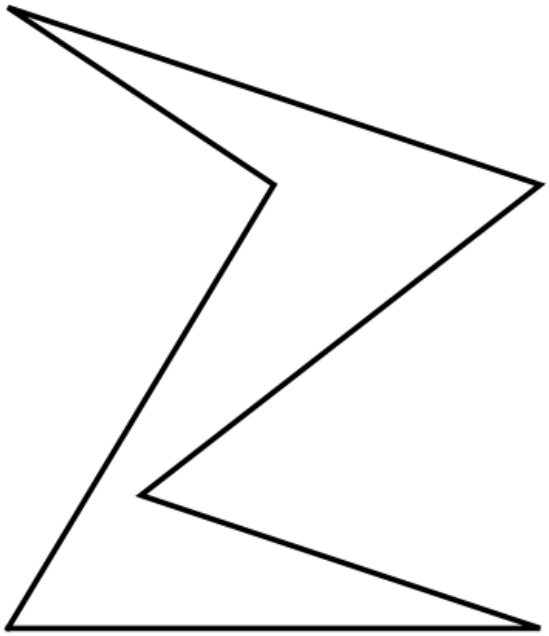
Sometimes one camera suffices for nonconvex galleries. See for example the three following galleries. Where do you have to place the camera? Is there another corner that also works?





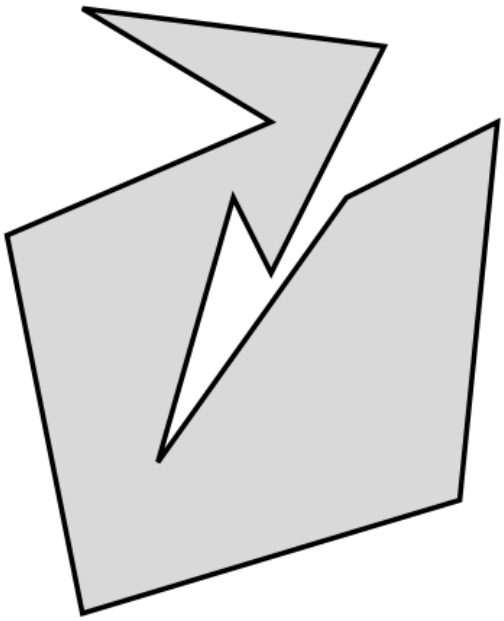


- Experiment that one camera does not suffice to watch the whole gallery in the following picture: for each of the six corners, draw the region watched by a camera located at this corner.

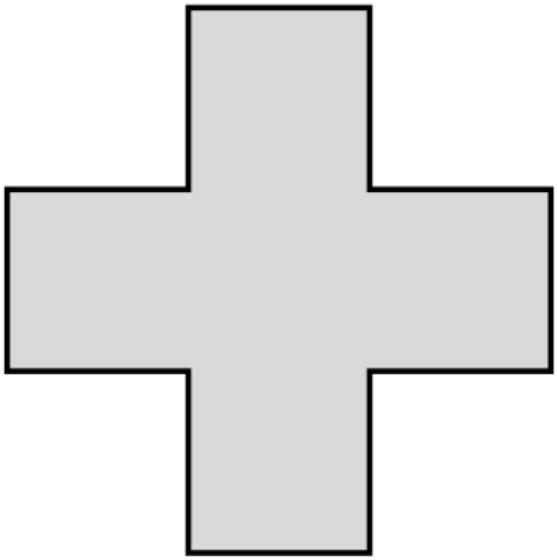


- However, two cameras suffice to watch the same gallery. Find all locations where you can place them.

- (More difficult) Show that two cameras suffice for this gallery.



- One camera suffices for this gallery. Can you find where it should be placed?



- Draw your own galleries and place a minimal number of cameras to watch the whole gallery.

Reference:
If you want to know more about the Art Gallery problem and experiment more, look at our other activity "The Art Gallery and the Fortress Problem". You'll also find more background material and resources there.

Create and Share!
Share the participant's galleries you created using the hashtags **#idm314gallery** and **#idm314**.

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